

(1) Simplify  $12 \pmod{5}$

$$2 \pmod{5}$$

(2) Simplify  $37 \pmod{15}$

$$7 \pmod{15}$$

(3) Simplify  $121 \pmod{9}$

$$4 \pmod{9}$$

(4) Simplify  $49 \pmod{3}$

$$1 \pmod{3}$$

(5) If  $12^3 \pmod{7} = 6$ , what is  $12^9 \pmod{7}$ ?

$$\begin{aligned} 12^9 &= 12^3 \cdot 12^3 \cdot 12^3 \pmod{7} \\ &= 6 \cdot 6 \cdot 6 \pmod{7} = 216 \pmod{7} = \boxed{6 \pmod{7}} \end{aligned}$$

(6) If  $15^4 \pmod{6} = 3$ , what is  $15^8 \pmod{6}$ ?

$$\begin{aligned} 15^8 &= 15^4 \cdot 15^4 \pmod{6} \\ &= 3 \cdot 3 \pmod{6} = 9 \pmod{6} = \boxed{3 \pmod{6}} \end{aligned}$$

(7) If  $9^6 \pmod{4} = 1$ , what is  $9^{13} \pmod{4}$ ?

$$\begin{aligned} 9^{13} &= 9^6 \cdot 9^6 \cdot 9^1 \pmod{4} \\ &= 1 \cdot 1 \cdot 9 \pmod{4} = 9 \pmod{4} = \boxed{1 \pmod{4}} \end{aligned}$$

(8) If  $18^6 \pmod{15} = 9$ , what is  $18^{13} \pmod{15}$ ?

$$\begin{aligned} 18^{13} &= 18^6 \cdot 18^6 \cdot 18^1 \pmod{15} \\ &= 9 \cdot 9 \cdot 18 \pmod{15} = 1458 \pmod{15} = \boxed{3 \pmod{15}} \end{aligned}$$

(9) Find the additive inverse for  $19 \pmod{26}$ .

$$\overline{19} \pmod{26} = (26 - 19) \pmod{26} = \boxed{7 \pmod{26}}$$

(10) Find the additive inverse for  $37 \pmod{26}$ .

$$\overline{37} \pmod{26} = \overline{11} \pmod{26} = (26 - 11) \pmod{26} = \boxed{15 \pmod{26}}$$

(11) Find the additive inverse for  $49 \pmod{21}$ .

$$\overline{49} \pmod{21} = \overline{7} \pmod{21} = (21 - 7) \pmod{21} = \boxed{14 \pmod{21}}$$

(12) Simplify  $-13 \pmod{25}$

$$\overline{13} \pmod{25} = (25 - 13) \pmod{25} = \boxed{12 \pmod{25}}$$

(13) Simplify  $-127 \pmod{15}$

$$\overline{127} \pmod{15} = \overline{7} \pmod{15} = (15 - 7) \pmod{15} = \boxed{8 \pmod{15}}$$

(14) Simplify  $-147 \pmod{21}$

$$\overline{147} \pmod{21} = \overline{0} \pmod{21} = (21 - 0) \pmod{21} = 21 \pmod{21} = \boxed{0 \pmod{21}}$$

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

(15) A Caesar Cipher is a special case of the Shift Cipher. What is  $\Delta$  for a Caesar Cipher?

$$\Delta = 3$$

(16) Using a Shift Cipher with  $\Delta = 7$ , encrypt the word TUGBOAT.

T	U	G	B	O	A	T
$\square = 20$	21	7	2	15	1	20
$\Delta = 7$	7	7	7	7	7	7
$\boxtimes = 27=1$	$28=2$	14	9	22	8	$27=1$
A	B	N	I	V	H	A

ABNIVHA

(17) Using a Shift Cipher with  $\nabla = 17$ , decrypt the word JRAYUJWN.

J	R	A	Y	U	J	W	N
$\boxtimes = 10$	18	1	25	21	10	23	14
$\Delta = 17$	17	17	17	17	17	17	17
$\square = 27=1$	$36=9$	18	$42=16$	$36=12$	$27=1$	$40=14$	$31=5$
A	I	R	P	L	A	N	E

AIRPLANE

(18) Use a Shift Cipher with the English Alphabet.

(a) if  $\Delta = 18$ , find  $\nabla$ .

$$26 - 18 = 8 = \nabla$$

(b) if  $\nabla = 3$ , find  $\Delta$ .

$$26 - 3 = 23 = \Delta$$

(19) A Vigenère Cipher is being used with the following shifts:

$$\Delta_1 = 2, \Delta_2 = 12, \Delta_3 = 21, \Delta_4 = 5$$

What Keyword is being used for this Vigenère Cipher?

2 12 21 5  
B L U E

(20) A Vigenère Cipher is being used with the following shifts:

$$\Delta_1 = 16, \Delta_2 = 21, \Delta_3 = 18, \Delta_4 = 16, \Delta_5 = 12, \Delta_6 = 5$$

What Keyword is being used for this Vigenère Cipher?

16 21 18 16 12 5  
P U R P L E

(21) Use the Keyword BOOK to encrypt the word LIBRARY using a Vigenère Cipher.

B	O	O	K	L	I	B	R	A	R	Y
$\Delta = 2$	15	15	11	$\square = 12$	9	2	18	1	18	25
				$\Delta = 2$	15	15	11	2	15	15
				$\boxtimes = 14$	24	17	$29=3$	3	$33=7$	$40=14$
				N	X	Q	C	C	G	N

NXQCCGN

(22) Use the Keyword CATS to decrypt the word IPIMEBFE using a Vigenère Cipher.

C	A	T	S	I	P	I	M	E	B	F	E
$\Delta = 3$	1	20	19	$\boxtimes = 9$	16	9	13	5	2	6	5
$\nabla = 23$	25	6	7	$\nabla = 23$	25	6	7	23	25	6	7
				$\square = 32=6$	$41=15$	15	20	$28=2$	$27=1$	12	12

F O O T B A L L